

**Exercise 19**

Prove that  $\frac{d}{dx}(\cot x) = -\csc^2 x$ .

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**Solution**

Use the quotient rule to differentiate the function.

$$\begin{aligned}\frac{d}{dx}(\cot x) &= \frac{d}{dx} \left( \frac{\cos x}{\sin x} \right) \\ &= \frac{\left[ \frac{d}{dx}(\cos x) \right] (\sin x) - \left[ \frac{d}{dx}(\sin x) \right] (\cos x)}{(\sin x)^2} \\ &= \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x} \\ &= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} \\ &= -\frac{1}{\sin^2 x} \\ &= -\csc^2 x\end{aligned}$$